

Bounds on scalar masses in two Higgs doublet models

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Abstract. A thorough analysis of stability and perturbativity bounds is performed in several versions of the two-Higgs doublet model, for both CP-conserving and spontaneously broken CP minima. LEP results further aid in establishing very strict constraints on the mass of the lighter Higgs particle.

1 Introduction

Despite the great successes of the Standard Model (SM) of particle physics, it leaves many unanswered questions, such as the origin of matter-antimatter asymmetry; although the SM does contain a CP-violating parameter in the CKM matrix, and violates baryon number, it is generally accepted that it does not lead to baryogenesis sufficient to explain the observed asymmetry. One of the simplest extensions of the SM, which tries to solve this problem, is the two-Higgs doublet model (2HDM) [1], wherein a second Higgs doublet is added to the theory. The spectrum of scalar particles becomes richer and, for some realisations of the model, spontaneous breaking of the CP symmetry is possible. The 2HDM presents some challenges, though: except in supersymmetric models, the quartic interactions between the scalar doublets are not theoretically constrained, and increase substantially the number of free parameters. As a consequence the predictive power of the model is reduced. Any tool available to constrain the parameter space is thus of great interest. In this paper we will take a closer look at the requirements of stability and perturbativity of the model. Namely, we will analyse their impact on the several possible incarnations of the model; including versions involving the imposition of two types of global symmetries, which eliminate several unknown parameters. For various reasons, it may be of interest to break those symmetries softly, by the introduction of quadratic coefficients in the potential. The possible different vacua of the model - minima which spontaneously break CP or preserve it - require a separate stability and perturbativity analysis, which we will perform. This paper is organised as follows: in Section 2 we will briefly review the basic notions about the 2HDM scalar potential and the requirements of stability and perturbativity for a range

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of renormalisation scales. This will lead to the computation of the one-loop β -functions of the model, (previously given in Ref. [3]). In sections 3 to 6 we will apply the stability and perturbativity bounds to the several realisations of the 2HDM: models with a discrete Z_2 or global $U(1)$ symmetries and their softly broken counterparts; within these, we will consider the possible cases of minima with spontaneously broken CP, or unbroken CP; and finally we will also consider the most general CP-conserving 2HDM potential. In all cases, we will endeavour to obtain bounds on the masses of the scalar particles, and use the latest experimental results on Higgs searches from LEP [4] to further constrain the potential's parameter space. Details of the β -function calculation are given in Appendix A, following a simple and pedagogical approach which may be of interest for readers unfamiliar with it; and in Appendix B we make some remarks about the renormalisation group invariance of basis-invariant conditions on the couplings.

2 The 2HDM potential

The 2HDM potential [1] involves two Higgs doublets with hypercharge $Y = 1$, Φ_1 and Φ_2 , and is invariant under the gauge symmetries of the standard model, $SU(3)_C \times SU(2)_W \times U(1)_Y$. The most general potential one can build with these two doublets, following the conventions of [5], is given by

$$\begin{aligned}
V = & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left(m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.} \right) + \\
& \frac{\lambda_1}{2} \left(\Phi_1^\dagger \Phi_1 \right)^2 + \frac{\lambda_2}{2} \left(\Phi_2^\dagger \Phi_2 \right)^2 + \lambda_3 \left(\Phi_1^\dagger \Phi_1 \right) \left(\Phi_2^\dagger \Phi_2 \right) + \lambda_4 \left(\Phi_1^\dagger \Phi_2 \right) \left(\Phi_2^\dagger \Phi_1 \right) + \\
& \left\{ \frac{\lambda_5}{2} \left(\Phi_1^\dagger \Phi_2 \right)^2 + \left[\lambda_6 \left(\Phi_1^\dagger \Phi_1 \right) + \lambda_7 \left(\Phi_2^\dagger \Phi_2 \right) \right] \left(\Phi_1^\dagger \Phi_2 \right) + \text{h.c.} \right\} , \tag{1}
\end{aligned}$$

where the couplings $\{m_{12}^2, \lambda_5, \lambda_6, \lambda_7\}$ are in general complex. In all, this potential has at most 14 real parameters. If one defines the CP transformation of the scalar fields as $\Phi_1 \rightarrow \Phi_1^*$, $\Phi_2 \rightarrow \Phi_2^*$, and requires that the potential above preserves this symmetry, all the parameters become necessarily real. The number of free real parameters is thus reduced to 10, or in fact 9, with an appropriate choice of basis for the scalar doublets. Namely, a given linear combination of Φ_1 and Φ_2 will always diagonalise the quadratic terms in the fields in Eq. (1). For all that follows, we will consider that the potential does not break CP explicitly, and thus all parameters are taken as real.

In general both Φ_1 and Φ_2 could have distinct Yukawa couplings to up-type quarks, down-type quarks and leptons. However, these generic Yukawa couplings would induce flavour-changing neutral currents (FCNC) in the theory. These have to be kept in check, either by imposing severe bounds on the size of the model's parameters or, more elegantly, by imposing symmetries upon it. Namely, a discrete Z_2 symmetry [6] or a global $U(1)$ [7] will prevent any FCNC from arising. This can be accomplished in several ways, but the choice we will make in this paper is to have only Φ_1 coupling to fermions. The results for the β -functions of the model are easily generalised to other situations, by means of the techniques detailed in Appendix A. In what follows we will, however, retain only the top quark Yukawa coupling, as the remaining ones will be too small to have any meaningful effect on the analysis we will perform. With this assumption we present in Appendix A the one loop β -functions for the theory defined by Eq. (1). Expressions for the β -functions for general models may be found in the literature [8], and the explicit expressions including the $\lambda_{6,7}$ contributions were given in Ref. [3]. Even with FCNC-preventing symmetries imposed upon it, the 2HDM potential has a great number of free parameters - 7 or more of them - a fact which severely curtails its predictive power. Any tools which help in limiting this vast parameter space are thus welcome. One way to limit the values

of the quartic couplings in Eq. (1) is by observing that general values for the λ_i do not guarantee that the potential is bounded from below (BFB). In fact, lest one requires that the quartic terms in Eq. (1) do not tend to minus infinity for any direction in field space, we will have no guarantees that the potential can have a stable minimum. For potentials where $\lambda_6 = \lambda_7 = 0$, Ivanov [9] has proven that the 2HDM potential is bounded from below if and only if the following conditions are obeyed:

$$\begin{aligned} \lambda_1 &> 0 & \lambda_3 &> -\sqrt{\lambda_1 \lambda_2} \\ \lambda_2 &> 0 & \lambda_3 + \lambda_4 - |\lambda_5| &> -\sqrt{\lambda_1 \lambda_2} \quad . \end{aligned} \quad (2)$$

These conditions have been widely used in the literature and assumed to be only *necessary* ones, but they are also in fact *sufficient*. The work of [9] gives, in principle, all necessary and sufficient conditions to have the potential Eq. (1) bounded from below even in the case $\lambda_6 \neq 0$, $\lambda_7 \neq 0$, but the relations one could derive in this situation are extremely complicated, and not at all clear. See also [10]. In [16] *necessary* conditions involving λ_6 and λ_7 were derived, and we will use them in the following work:

$$2|\lambda_6 + \lambda_7| < \frac{\lambda_1 + \lambda_2}{2} + \lambda_3 + \lambda_4 + \lambda_5 \quad . \quad (3)$$

As explained above, the conditions Eqs. (2) and (3) ensure the stability of the tree scalar potential. To be sure of a viable vacuum, however, one must take into account the effect of radiative corrections, and the related fact that the λ_i depend on the renormalisation scale μ .

Let us first review this important issue in the context of a theory with a single scalar field, the real scalar ϕ^4 -model, with

$$V_{cl} = \frac{m^2}{2}\phi^2 + \frac{\lambda}{4!}\phi^4 \quad (4)$$

so that

$$V = V_{cl} + \frac{V_{cl}''(\phi)^2}{64\pi^2} \ln\left(\frac{V_{cl}''(\phi)}{\mu^2}\right) + \dots \quad (5)$$

Let us suppose for simplicity that $m^2 > 0$. For what values of ϕ can we reliably calculate V ? Suppose we have chosen a RG scale $\mu \sim m$ and that λ is small on that scale so that perturbation theory in λ is believable. Then evidently we can calculate V as $\phi \rightarrow 0$ by simply retaining $\mu \sim m$ since the one loop correction is obviously small. Thus the origin remains a minimum, as was the case for the tree potential. But what about $\phi \gg m$? The one-loop correction now becomes large, because of the logarithm, so that one must improve on this perturbation expansion. RG improvement amounts, in fact, to exploiting the freedom to choose the renormalisation scale to take $\mu^2 \sim V_{cl}''(\phi)$, or $\mu \sim \phi$ for large ϕ . Then to a good approximation at large ϕ we will have

$$V = \frac{\lambda(\phi)}{4!}\phi^4 \quad (6)$$

and this will be perturbatively believable as long as $\lambda(\phi)$ is small. Now in this simple model λ becomes large at large scales, approaching a Landau pole, and so perturbation breaks down eventually in spite of our RG improvement. Thus we cannot say what form the potential takes at sufficiently large ϕ .

In a more complicated theory there are two main issues to take into consideration. Firstly, if the potential depends on more than one scalar field, it is not immediately obvious in which directions in field space we will be able to describe the large-field potential, since we have only one scale at our disposal¹. Secondly, the behaviour of the $\lambda_i(\phi)$ for large ϕ may be quite

¹For an attempt to generalise the RG discussion to incorporate more than one μ see Ref. [17].

different from that in the simple ϕ^4 -model. In particular, in the 2HDM the large size of the top quark Yukawa coupling, and the sign of its contribution to the β -function of λ_1 (see Eq. (44) in Appendix A) drives down the value of that quartic coupling as one goes up in renormalization scale. If the starting point of λ_1 is sufficiently small, λ_1 may become negative at a given high value of μ , which would mean that any minimum present for low renormalization scales would in fact be unstable - the potential would either be unbounded from below or develop a much deeper minimum at large ϕ_1 ².

The approach we shall take is to simply assume that the stability conditions Eqs. (2),(3) must hold at all renormalisation scales μ up to the (putative) gauge unification scale $M_U = 10^{15}$ GeV. This will clearly be sufficient to produce a potential bounded from below. Requiring the *stability* of the scalar potential at all scales will thus, typically, impose *lower* bounds on the values of its quartic couplings.

Another way of limiting the values of the λ_i is by requiring that they remain small enough for perturbative believability at high scales. Hence, if the initial values of λ_i are too large, their β -functions will be positive and their renormalization scale evolution will drive them to ever higher values. Requiring that the λ_i remain small at all scales will thus impose *upper* bounds on their values. How small should “small” be? Here we enter a somewhat arbitrary region, but requiring that all λ_i remain less than 10 at all renormalization scales seems a reasonable requirement.

We will therefore impose both stability and perturbativity bounds on the quartic parameters of the 2HDM at all scales between the weak scale M_Z and M_U . Such analyses have been made before, in many works: these ideas were applied to the SM [18], SUSY models [19] and also to a simple 2HDM [20]. In this work we are interested in studying the differences that the application of these bounds will have on the several possible two-Higgs doublet models, and on the several possible vacua therein possible.

3 Model with Z_2 symmetry

One of the symmetries that rids the potential Eq. (1) of FCNC was first proposed by Glashow, Weinberg and Paschos [6], and consists of a simple Z_2 transformation in the fields: $\Phi_1 \rightarrow -\Phi_1$, $\Phi_2 \rightarrow \Phi_2$. By carefully choosing similar transformations for the fermionic fields it is possible to eliminate the existence of FCNCs by having, for instance, only Φ_1 couple to the fermions. This symmetry simplifies Eq. (1), namely setting to zero several of the couplings: $m_{12} = \lambda_6 = \lambda_7 = 0$. Then the BFB conditions of Eq. (2) are, in this case, necessary and sufficient.

Our procedure was as follows: we generated many thousands of combinations of quartic parameters of the 2HDM. The couplings were generated with magnitudes between 10^{-3} and 10, allowing different couplings to have different orders of magnitude and to be negative if allowed. Spontaneous symmetry breaking occurs when the doublets acquire vacuum expectation values such that

$$\langle \Phi_1 \rangle = \begin{pmatrix} 0 \\ v_1 \end{pmatrix} \quad , \quad \langle \Phi_2 \rangle = \begin{pmatrix} 0 \\ v_2 \end{pmatrix} \quad . \quad (7)$$

We thus generated values for the vevs $\{v_1, v_2\}$ such that $v_1^2 + v_2^2 = v^2$, with $v = 246/\sqrt{2}$ GeV. With the vevs and all quartic couplings, it is simple to use the stationarity conditions of the model and determine the quadratic parameters m_{11}^2 and m_{22}^2 . At this point, we have a full set of parameters for the potential. By analysing the model’s squared scalar mass matrices

²In fact, in the type of theory we consider here, the latter is generally the case because the positive contribution of the gauge coupling contributions to their β -functions causes λ_i to recover to positive values at yet higher scales.

(expressions for which may be found, for instance, in [16]), we can ensure that each combination of parameter values under consideration is indeed a Normal minimum of the 2HDM.

We then analysed the RG evolution of the quartic couplings for each “point” of parameter space and checked whether they obeyed the stability and triviality bounds described above, between M_Z and M_U . In Fig. 1 we see the result of this procedure. In this plot we show the

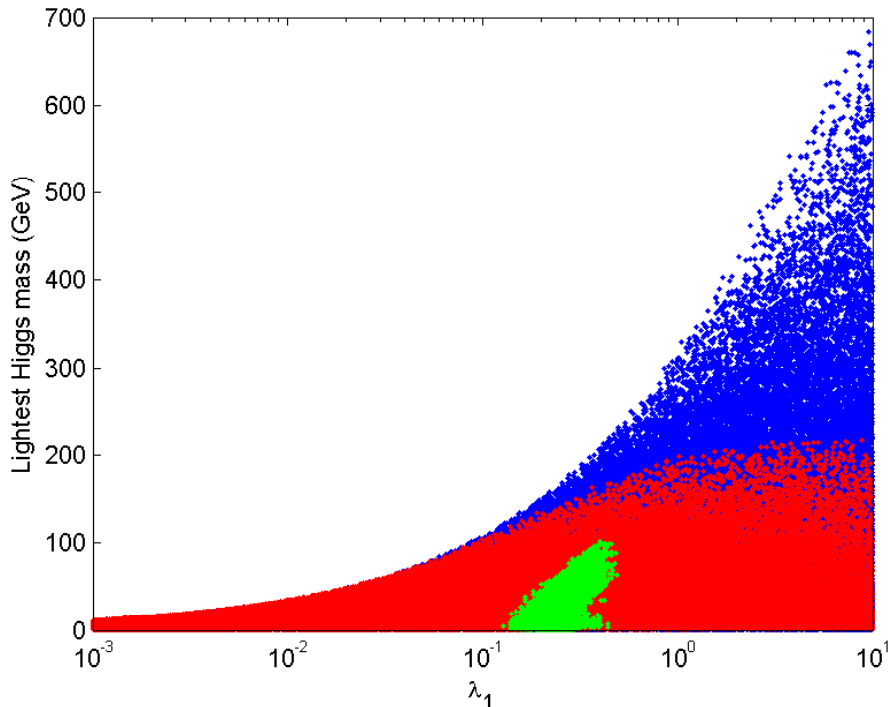


Figure 1: Results of scan of the 2HDM potential with Z_2 symmetry.

lightest CP-even Higgs mass versus the value of the λ_1 coupling. The colours are interpreted as:

- The red (medium) points represent those combinations of 2HDM parameters for which the stability conditions of Eq. (2) were violated somewhere between M_Z and M_U .
- The blue (dark) points represent all of the parameter combinations which passed the stability conditions of Eq. (2), but for which no triviality conditions were set.
- Finally, the green (lightest) points are a subset of the blue ones - those for which the triviality conditions are obeyed at all scales between M_Z and M_U ³.

As we see, the combination of stability and triviality conditions narrows the “allowed” range of λ_1 immensely - the only values which “survive” are in the interval $0.24 < \lambda_1 < 0.91$. The remaining couplings are likewise constrained in similar intervals, of identical order of magnitude. This also limits high values for the Higgs scalar masses. In fact, if one analyses the full spectrum of scalar particles, one finds that after the stability and triviality requirements the masses are

³Notice that there are many blue points “between” the red ones, even if in the plot they are “covered” by the red points. This means that not all points with Higgs mass below ~ 200 GeV are excluded on the basis of the stability conditions, and that the green region is indeed a subset of the blue one. This is not a contradiction - this parameter space includes seven different parameters, and this plot is only varying two. We can therefore have a “rejected” point and an “accepted” one occupying the same place in the plot.

bounded by (roughly):

$$\begin{aligned}
m_h &< 102 \text{ GeV} \\
121 &< m_H < 199 \text{ GeV} \\
m_A &< 163 \text{ GeV} \\
m_{H^\pm} &< 160 \text{ GeV} \quad .
\end{aligned} \tag{8}$$

These results, specially those pertaining to the charged Higgs mass, are in agreement with previous works (see the last reference of [20]). Unless explicitly stated, no lower bounds were found for these masses, the exception being the heaviest neutral scalar H . These bounds do not preclude very low Higgs masses, then. In fact, current experimental data does not forbid light neutral 2HDM scalars. The best bounds on Higgs masses arise from the latest LEP results [4] and the analysis of associated production of a Z boson with the lightest 2HDM CP-even scalar, through the triple vertex ZZh . In the 2HDM, the coupling associated with this vertex is equal to its SM value, multiplied by $\sin(\alpha - \beta)$, where $\tan \beta = v_2/v_1$ and α is the usual mixing angle for the CP-even scalar mass matrix. If this coupling is small - meaning, if $\sin(\alpha - \beta)$ is small - then the mass of the scalar particle can be also small and still have escaped detection at LEP. This was explored recently for both SUSY models and one version of the 2HDM (see, for instance, [21]). In fact, the cross section for $e^+e^- \rightarrow Zh$ production in the 2HDM is related to that of the SM by

$$\sigma^{2HDM}(e^+e^- \rightarrow Zh) = \sin^2(\alpha - \beta) \sigma^{SM}(e^+e^- \rightarrow Zh) \quad . \tag{9}$$

This relation is valid for any type of 2HDM model with a CP-conserving vacuum. The LEP results impose severe constraints on the size of the ratio $\sigma^{2HDM}/\sigma^{SM}$ which, considering the previous equation, translate as constraints on $\sin^2(\alpha - \beta)$. In Fig. 2 we plot the value of $\sin^2(\alpha - \beta)$ against the mass of the lightest Higgs boson, for the subset of parameter space which survived the stability and triviality bounds (the green (light) points from Fig. 1). The red (continuous) line drawn in this plot corresponds to the experimental bound coming from the LEP searches [4]. Only the points below this line are allowed. We therefore see that the majority of the points which survived the triviality and stability analysis are already excluded on experimental grounds. Although it seems possible to generate high masses for the lightest Higgs particle, this plot clearly indicates that lower masses are preferred. Indeed, a rough upper bound of ~ 55 GeV can be established from these data.

Nevertheless, caution must be urged. However large our sampling of the parameter space, it does not cover all regions of it. Also, these results are sensitive to the input top quark mass, which still has some uncertainty, according to the most recent Tevatron results [23]: the CDF and D0 combined value is $M_t = 173.1 \pm 1.3$ GeV. The physical mass corresponds to the pole of the propagator, and its relation to the Yukawa coupling h_t and the vev v_1 is given, up to one loop, by

$$M_t^{pole} = h_t v_1 \left[1 + \left(4 - 3 \ln \left(\frac{h_t^2 v_1^2}{\mu^2} \right) \frac{\alpha_S}{3\pi} \right) \right] \quad , \tag{10}$$

where we are only taking the most significant corrections, those from QCD. μ is the renormalization scale considered, and all quantities in the formula above are evaluated at that scale. The results presented thus far (and elsewhere in this paper) assume a top pole mass of 173 GeV. We verified what changes occur if we varied the top pole mass by 2 GeV in either direction (a conservative variation). The bounds shown in Eq. (8) that change by variation of M_t^{pole} are shown in Table 1. The lower bound on the heaviest CP-even scalar is the one that changes the most. In fact, that lower bound correspond to small values of v_2 , for which one of the masses, h or H , is essentially proportional to λ_1 . The lower bounds for m_H presented in the table above

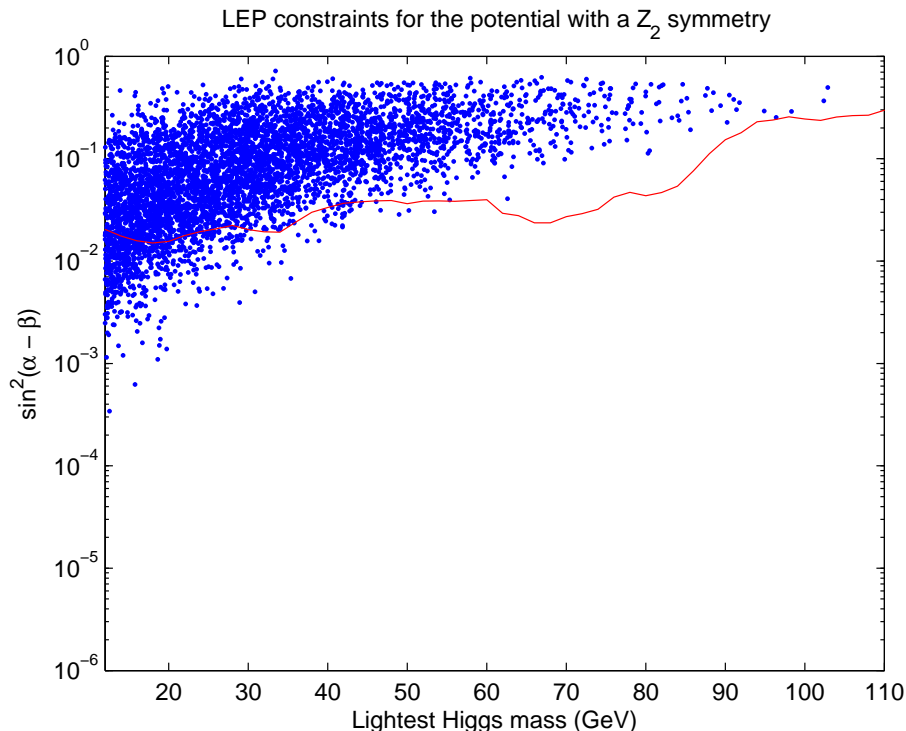


Figure 2: $\sin^2(\alpha - \beta)$ versus the mass of the lightest Higgs boson, for a potential with a Z_2 symmetry.

| Pole mass (GeV) | $M_t^{pole} = 171$ | $M_t^{pole} = 173$ | $M_t^{pole} = 175$ |
|----------------------|--------------------|--------------------|--------------------|
| Mass bounds (GeV) | $m_h < 100$ | $m_h < 102$ | $m_h < 102$ |
| | $m_H > 119$ | $m_H > 121$ | $m_H > 129$ |
| | $m_A < 165$ | $m_A < 163$ | $m_A < 162$ |

Table 1: Bounds on scalar masses in function of the value of the top quark pole mass.

correspond to the lower allowed values for λ_1 , which obviously change when the top pole mass is varied. The uncertainty on the top pole mass is thus relevant, and needs to be factored in evaluating whatever bounds we will present in this work.

Still, the results shown in Fig. 2 clearly indicate that the 2HDM with a Z_2 symmetry is already severely constrained by the simultaneous requirements of stability, triviality and compliance with existing experimental results. The pole mass dependence is of the order of ~ 5 GeV around the central values at most, and will not drastically change those conclusions.

3.1 The case $v_2 = 0$

For the 2HDM with an unbroken Z_2 symmetry, the minimisation conditions admit a different type of solution than the one we have been considering: to wit, a vacuum where one of the fields Φ has a vanishing expectation value. These models were first proposed in [11] and have been studied before, in many different contexts. For instance, in ref. [12], one such model was used to show that it was possible to have neutrino mixing even without massive neutrinos. In [13] the model was used to explain low neutrino masses as a loop effect. In general these models are

the basis of the so-called “inert Higgs” theories [14], which have excellent scalar candidates for dark matter [15].

Since in our models only Φ_1 couples to the fermions, we should therefore study the case where $v_2 = 0$, and verify what changes occur in the bounds we have deduced ⁴. The expressions for the squared scalar masses in these models are extremely simple, namely

$$\begin{aligned} m_{h_1}^2 &= 2\lambda_1 v^2 & m_{h_2}^2 &= m_{22}^2 + (\lambda_3 + \lambda_4) v^2 \\ m_{H^\pm}^2 &= m_{22}^2 + \lambda_3 v^2 & M_A^2 &= m_{h_2}^2 \quad , \end{aligned} \quad (11)$$

where $m_{h_1}^2$ and $m_{h_2}^2$ are the two CP-even scalar masses, the lightest of which will be h , the heaviest H (depending on the parameters, though, we cannot *a priori* guarantee which of h_1 and h_2 is the lightest state). In the model of the previous section, the minimisation conditions ensured a strong bond between the values of the squared parameters, m_{11}^2 and m_{22}^2 , the scale v^2 and the values of the quartic parameters λ_i . With $v_2 = 0$, though, despite the fact that m_{11}^2 is fixed such that $m_{11}^2 = -\lambda_1 v^2$, the parameter m_{22}^2 is unconstrained and as such can be as large as one wishes. Thus, we expect the upper bounds on most of the masses written above to be much larger than before. In fact, once the stability and triviality analysis is concluded, we obtain

$$\begin{aligned} m_h &< 235 \text{ GeV} \\ m_H &> 120 \text{ GeV} \quad . \end{aligned} \quad (12)$$

The only upper bound that remains is that on m_h , certainly due to the h_1 state, which is directly tied to the severely constrained λ_1 coupling. Because the CP-even 2×2 mass matrix is diagonal, the mixing angle α has only two possible values: 0 and $\pi/2$. As such (and since in this model the angle β is equal to zero), the LEP results have no impact on the model:

- If $\alpha = 0$, the coupling of h to the Z boson vanishes. This occurs for a large range of masses, from very low Higgs masses to high ones. In any case, the LEP data do not provide any constraints, since this lightest scalar does not couple to the Z and as such could not have been observed at LEP.
- If $\alpha = \pi/2$, the coupling of h to Z is identical to that of the SM. However, this case only occurs for values of the Higgs mass larger than 116 GeV. The LEP constraints, as can be observed from fig. 2, are only valid for Higgs masses inferior to about 110 GeV. As such, the case $\alpha = \pi/2$ is also not constrained by the LEP data.

4 Model with softly broken Z_2 symmetry

If one adds to the Z_2 potential a term of the form $m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}$ the discrete symmetry is softly broken. However, no FCNCs arise from this soft breaking. The main reason why doing this should be of interest is quite simple - with this soft breaking term the potential can now have two types of interesting minima: (a) “normal” ones, which preserve CP, and for which the doublet’s vacuum expectation values, as before, have the form of Eq. (7); and (b), minima for which CP is spontaneously broken, the doublets developing vevs of the form

$$\langle \Phi_1 \rangle = \begin{pmatrix} 0 \\ v_1 \end{pmatrix} \quad , \quad \langle \Phi_2 \rangle = \begin{pmatrix} 0 \\ v_2 + iv_3 \end{pmatrix} \quad . \quad (13)$$

⁴We thank the referee for bringing this point to our attention.

In order to have minima of phenomenological interest, the CP vevs must obey $v_1^2 + v_2^2 + v_3^2 = v^2 \text{ GeV}^2$. It has recently been proved [9] that these different minima cannot co-exist. The parameters of the potential either allow the normal minimum or a CP breaking one⁵. Thus, the parameter sets for each of these cases have to be different, and generated separately. The different vevs, however, do not directly affect the RGE running of the quartic couplings. Notice also that the existence of another free parameter (m_{12}^2) in the potential will certainly change the scalar masses.

4.1 Normal minima

For this case, the parameters of the potential were chosen such that the global minimum of the potential preserves CP. The range of allowed values for the λ_i was the same as before, and the soft breaking parameter m_{12}^2 is chosen so that all three mass squared parameters are of the order v^2 , and so that $|m_{12}^2| < 10v^2$. Once again, thousands of different parameter sets were generated, and then RGE analysed between the weak and unification scales. Again we found that only a narrow window of values of λ_1 survived the imposition of stability and triviality bounds. The results differ from those of the unbroken Z_2 model, the bounds found for the masses being given by

$$\begin{aligned} m_h &< 187 \text{ GeV} \\ 121 &< m_H < (\text{no upper limit}) \text{ GeV} \quad . \end{aligned} \tag{14}$$

As explained earlier, the quadratic parameters m_{11}^2 and m_{22}^2 are obtained from the stationarity conditions of the potential, once all other parameters have been generated. For the unbroken Z_2 potential, those included all the quartic couplings (limited by the triviality and stability requirements) and the vevs (limited by the requirement of their squared sum be equal to $(246/2)^2 \text{ GeV}^2$). Now, however, we have the extra quadratic parameter in the potential, which is not restricted and can make the masses larger than they can be in the unbroken Z_2 model. This justifies the lessening of the bounds we discover.

In Fig. (3) we again plot the effect of the LEP bounds; once again the allowed region is below (to the right of) the line. As we see, the soft-breaking term allows us to easily evade the experimental constraints.

4.2 CP breaking minima

As was shown in Ref. [16], the quartic parameters of the 2HDM potential need to obey a specific condition so that there is a CP-breaking minimum⁶. Thus, if the quartic parameters are necessarily different from those which generate Normal minima, one expects different results stemming from the stability and triviality bounds.

Notice, now, that there is no distinction between CP-even and CP-odd scalars, since CP is spontaneously broken - the complex vevs shown in Eq. (13) will cause a mixing between all neutral components of the doublets. We rename the neutral scalars as h_1 , h_2 and h_3 , in decreasing order of masses. After repeating the stability and triviality RG analysis, we found the following bounds for these scalar masses:

$$131 < m_{h_1} < 203 \text{ GeV}$$

⁵Charge breaking vacua might also occur but, as shown in [16], whenever a normal minimum exists in the 2HDM, the global minimum of the potential is normal and thus safe against charge breaking. Likewise, if a CP minimum exists, any charge breaking stationary point will necessarily be a saddle point.

⁶Namely, the quartic parameters must be such that the matrix B_{CP} defined in [16] be positive definite, and so that $\lambda_4 < \lambda_5$.

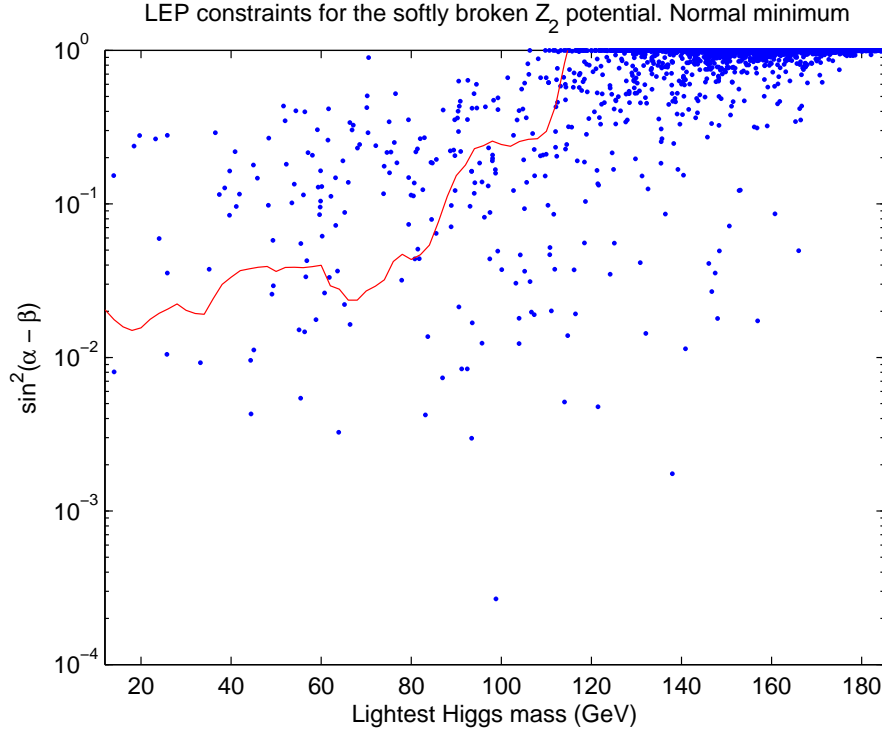


Figure 3: $\sin^2(\alpha - \beta)$ versus the mass of the lightest Higgs boson, for the normal minimum of a potential with a softly broken Z_2 symmetry.

$$\begin{aligned}
 6 < m_{h_2} < 160 \text{ GeV} \\
 m_{h_3} < 84 \text{ GeV} \\
 m_{H^\pm} < 158 \text{ GeV} \quad .
 \end{aligned} \tag{15}$$

Notice the very low upper bound on the lightest neutral scalar, and indeed in all of the scalars. Clearly the requirement of a CP-breaking vacuum “chooses”, from within the parameter space which survives the stability and triviality bounds, scalars with lower masses, which is quite different from what we saw occurring for this same model, for Normal minima. One might speculate that this phenomenon is somehow related to the Georgi-Pais theorem [26] according to which spontaneous breaking of CP via *radiative corrections* is always accompanied by scalars which are massless in the tree approximation (of course here we are considering tree-level CP breaking).⁷

The LEP results also have a substantial impact on the parameter space for these minima. But to apply them, we must first compute the coupling between the Z boson and the lightest Higgs scalar for a CP-breaking minimum. As was explained earlier, the complex vevs of Eq. (13) cause a mixing between CP-even and CP-odd scalars, so that the masses of the neutral scalars h_1 , h_2 and h_3 are the eigenvalues of a 4×4 matrix (the fourth eigenvalue is zero, corresponding to the Z would-be Goldstone mode G^0). Consequently there is now no single angle α which characterises the diagonalisation of this matrix, and thus the quantity $\sin(\alpha - \beta)$, which was the ratio between the $Z Z h$ coupling in the 2HDM and the SM, is no longer defined for CP-breaking minima. For the determination of the $Z Z h$ coupling, the relevant term in the Lagrangian stems

⁷We thank the referee for a comment on this point.

from the kinetic terms for the doublets, namely

$$\sum_{i=1}^2 (D_\mu \Phi_i)^\dagger (D^\mu \Phi_i) \rightarrow \frac{1}{8} g^2 \sec^2 \theta_W Z_\mu Z^\mu \sum_{i=1}^2 |\Phi_i|^2 . \quad (16)$$

Let us define the neutral component fields of the doublets as

$$\Phi_1^0 = R_1 + iI_1 \quad , \quad \Phi_2^0 = R_2 + iI_2 \quad . \quad (17)$$

The relationship between these fields and the mass-eigenstates h_1, h_2, h_3 and G^0 is given by a 4×4 unitary matrix A_{ij} , such that

$$\begin{pmatrix} R_1 \\ R_2 \\ I_1 \\ I_2 \end{pmatrix} = A \begin{pmatrix} h_1 \\ h_2 \\ h_3 \\ G^0 \end{pmatrix} . \quad (18)$$

Then, the terms in Eq. (16) which are directly proportional to the lightest Higgs, h_3 , will be given by

$$\frac{1}{4} g^2 \sec^2 \theta_W Z_\mu Z^\mu h_3 (A_{13}v_1 + A_{23}v_2 + A_{43}v_3) \quad (19)$$

so that the ratio of the ZZh coupling in a CP minimum and that of the SM is given by

$$g_{ZZh} = \frac{1}{v} (A_{13}v_1 + A_{23}v_2 + A_{43}v_3) \quad (20)$$

with $v = 246/\sqrt{2}$ GeV. Therefore, the generalisation of Eq. (9) is thus

$$\sigma_{CP \text{ minimum}}^{2HDM}(e^+e^- \rightarrow Zh) = g_{ZZh}^2 \sigma^{SM}(e^+e^- \rightarrow Zh) . \quad (21)$$

It is now a simple task to calculate the mass matrix for the neutral scalars, diagonalise it and obtain the matrix A and thus compute the value of g_{ZZh} for the points of parameter space which survived the stability and triviality bounds, comparing them with the LEP results. As we see from Fig. (4), the LEP results exclude a significant portion of the parameter space. (As before the allowed region is below the line). In fact, according to this plot, the highest value allowed for the lightest Higgs mass would be around 65 GeV. Comparing this plot to figs. 2 and 3, we see that the bounds we are imposing have very different consequences, depending on the model, or type of minimum, considered.

5 Model with a softly broken $U(1)$ symmetry

Another symmetry to eliminate FCNC in the original potential Eq. (1) is a simple global $U(1)$ transformation in the fields (accompanied by suitable fermion transformations) of the form $\Phi_1 \rightarrow \Phi_1, \Phi_2 \rightarrow e^{i\alpha} \Phi_2$. This symmetry simplifies Eq. (1), again setting to zero several of its couplings, $m_{12} = \lambda_5 = \lambda_6 = \lambda_7 = 0$. This symmetry is however *too* strong, in that it produces a zero mass axion⁸. To prevent that from happening one usually softly breaks this global $U(1)$ by re-introducing the m_{12}^2 term in the potential. Except for the fact that the remaining λ couplings are unrelated to one another, the resulting Higgs potential is similar to the MSSM one. The only types of minima this potential possesses are normal ones; CP breaking is impossible here (see also footnote 5).

⁸Unless one considers a vacuum with $v_2 = 0$, as in section 3.1, with analogous consequences.

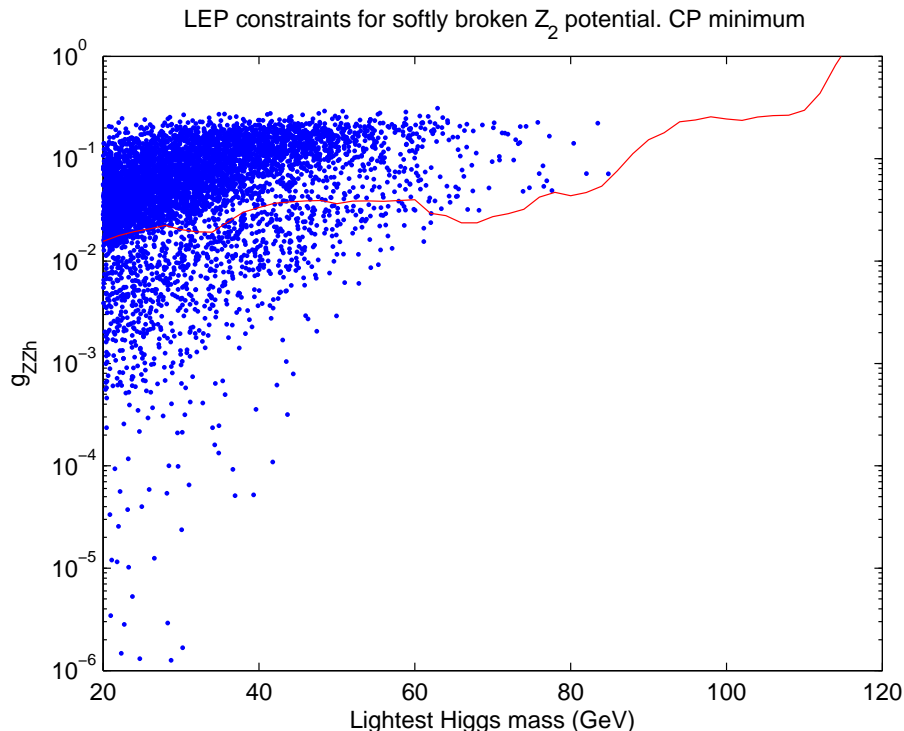


Figure 4: g_{ZZh}^2 versus the mass of the lightest Higgs boson, for the CP breaking minimum of a potential with a softly broken Z_2 symmetry.

Once again we generated thousands of parameter sets corresponding to normal minima and used the model's β -functions to verify whether the stability and triviality conditions were satisfied all the way up to M_U . In terms of that analysis, the only difference with the softly-broken Z_2 model studied in section 4.1 is the fact that for this model we must have $\lambda_5 = 0$, even after the $U(1)$ symmetry has been softly broken. As before, only a very narrow range of values of λ_1 survived the stability and triviality requirements. The bounds found for the scalar masses are now:

$$\begin{aligned} m_h &< 187 \text{ GeV} \\ 121 &< m_H < (\text{no upper limit}) \text{ GeV} . \end{aligned} \quad (22)$$

The LEP constraints do not affect the available parameter space as much as in the model with unbroken Z_2 symmetry, or for CP minima in the softly broken Z_2 model; the results are shown in Fig. (5). The results are quite similar to those obtained in section 4.1 for normal minima in the softly-broken Z_2 model, no doubt due to the presence of in both cases of the soft-breaking parameter m_{12}^2 .

6 The full CP conserving potential

What about the full potential of Eq. (1)? What do the triviality and stability bounds tell us about it? To perform this analysis we need the β -functions for this model, in terms of the “new” couplings λ_6 and λ_7 . These are to be found in Appendix A. As we have mentioned, in this case FCNC will, in general, occur.

Due to the presence of the new quartic couplings, the stability analysis needs to take into account Eq. (3), which involves λ_6 and λ_7 . And once more, as with the softly broken Z_2 model,

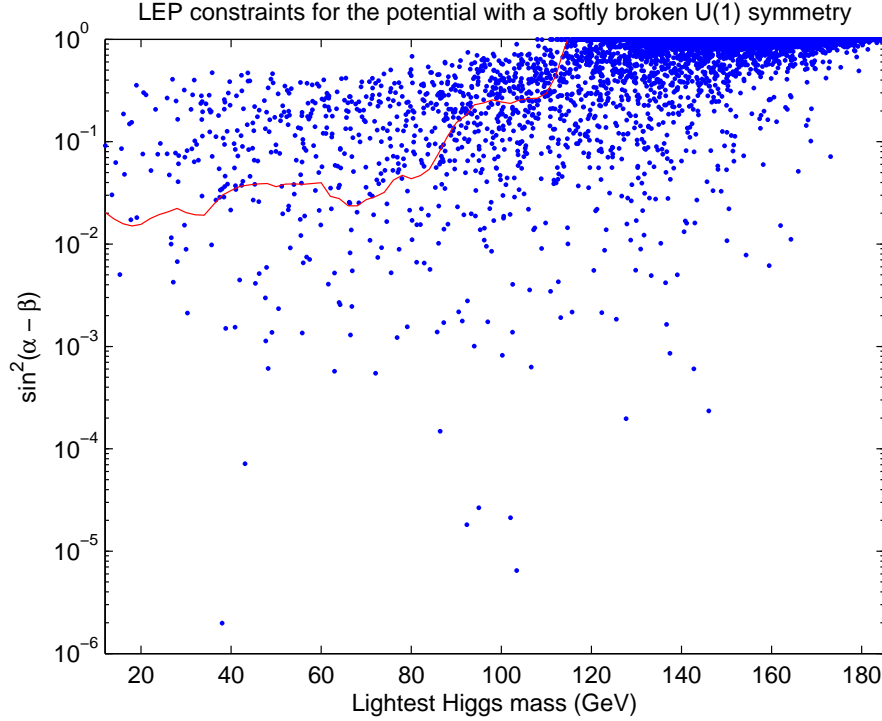


Figure 5: $\sin^2(\alpha - \beta)$ versus the mass of the lightest Higgs boson, for the CP breaking minimum of a potential with a softly broken $U(1)$ symmetry.

this potential can have minima with spontaneous CP breaking and Normal minima, though the same set of parameters cannot produce two such minima in coexistence. We thus obtain the following bounds for each type of minima: for Normal minima, the only bounds found were

$$\begin{aligned} m_h &< 187 \text{ GeV} \\ 121 &< m_H < (\text{no upper limit}) \text{ GeV} \end{aligned} \quad , \quad (23)$$

and for CP minima,

$$\begin{aligned} 131 &< m_{h_1} < 205 \text{ GeV} \\ 6 &< m_{h_2} < 166 \text{ GeV} \\ m_{h_3} &< 84 \text{ GeV} \\ m_{H^\pm} &< 158 \text{ GeV} \end{aligned} \quad . \quad (24)$$

As we can see, there are no great differences with the results obtained for the Normal minima of the softly broken Z_2 potential, or for the $U(1)$ model. The new parameters do not affect the bounds on the masses, nor do they change the qualitative difference between the bounds obtained for each type of minima: that the requirements of stability and triviality tend to “pick” lower scalar masses for CP minima than they do for Normal minima. Also, notice that the case $v_2 = 0$, discussed in section 3.1, is only possible for the Z_2 symmetric potential (in fact solutions with $v_2 = 0$ become possible as long as $m_{12}^2 = \lambda_6 = 0$, but without the full Z_2 symmetry these conditions would not be preserved by renormalisation). It corresponds to a completely different type of vacuum (one which preserves the Z_2 symmetry) for which the bounds we found for that case (eq. (12)) are unaffected.

The LEP constraints are again similar to those already shown, and we present them, for both minima, in Fig. 6. Once again, much of the available parameter space is excluded by the

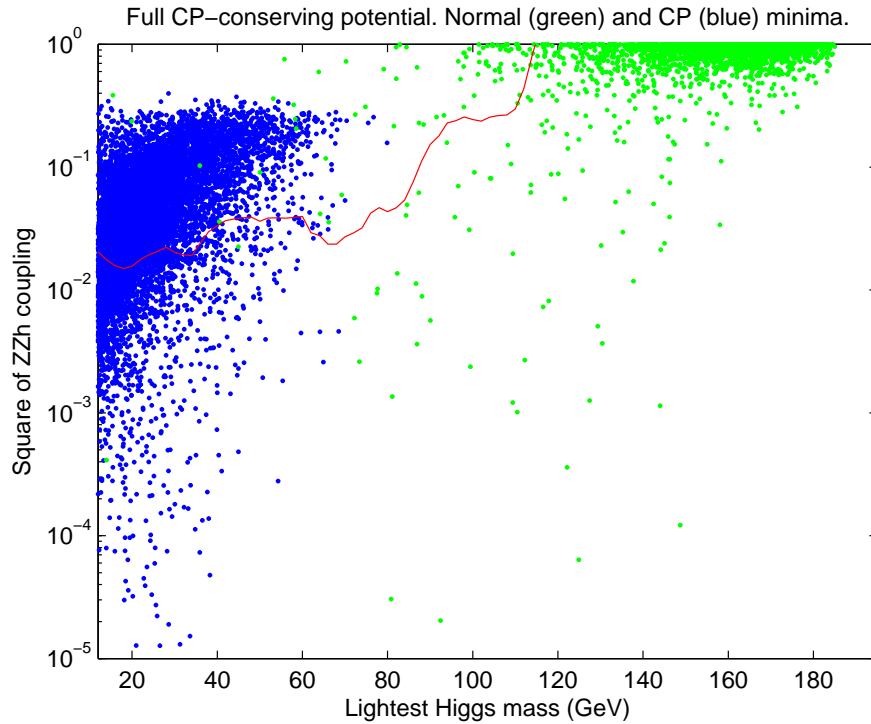


Figure 6: Squared ZZh coupling versus the mass of the lightest Higgs boson, for the full CP conserving potential. The green (light) points concern the Normal minima, the blue (dark) ones the minima which spontaneously break CP.

LEP data for the case of the CP minima. The lightest Higgs scalar, in that case, would have a rough upper bound of ~ 80 GeV.

7 Conclusions

We have performed a thorough analysis of the impact that the demands of stability and triviality have on the scalar masses of the 2HDM. We considered several possible incarnations of this model - models with Z_2 or $U(1)$ symmetries, with those symmetries softly broken or simply without them, and the different neutral vacua allowed in those theories. At the same time, we studied the impact of the LEP results on production of a light Higgs scalar on the parameter space of the model. Our results may be summarised as follows:

- The 2HDM potential with a Z_2 symmetry is very strongly constrained. Combining both theoretical and experimental constraints, the mass of the lightest neutral scalar should be less than about 55 GeV.
- The LEP restrictions are easily avoided in Normal minima of models with softly broken Z_2 or $U(1)$ symmetries, or in the full CP-conserving potential. In those models, the lightest CP-even neutral scalar is bound to be smaller than about 190 GeV. The heaviest CP-even neutral scalar is bound to be larger than about 120 GeV.
- The minima with spontaneous CP breaking which may occur in these models are heavily constrained. For these minima, the stability and triviality bounds affect the parameter space of these models in different ways than what occurs for the Normal minima. Those

bounds constrain very tightly *all* scalar masses, and the LEP results have an extremely strong impact on the surviving parameter space, eliminating most of it.

In order to perform this analysis, we needed the β -functions for the parameters λ_6 and λ_7 , which are given in Appendix A and were given previously in [3]. These β -functions allowed us to verify the validity of necessary conditions involving λ_6 and λ_7 between the weak and renormalization scales. Their usefulness, however, is not restricted to the studies presented here. In [27], for instance, basis-invariant conditions which ensure greater symmetry of the 2HDM potential were obtained. Several of those conditions involve the parameters λ_6 and λ_7 . To verify if these conditions are valid at all renormalisation scales, it will be necessary to employ the β -functions. We perform this analysis in Appendix B.

The parameters λ_6 and λ_7 are often omitted because in their presence there is no global symmetry which we can use to automatically prevent flavour changing neutral currents. We have avoided this issue by assuming that quarks couple to Φ_1 only, and anyway retaining only the top quark Yukawa coupling. Of course the absence of a symmetry to enforce this means that to pursue the question of FCNCs we would need to consider the effect of radiative corrections. In any event, we found the effect of $\lambda_{6,7}$ on our analysis to be limited - the analysis of the stability and triviality bounds for the full CP-conserving potential did not produce significantly different results from those obtained for the softly broken Z_2 or $U(1)$ models. That in itself, however, is an interesting result: if for whatever reason one wishes to work with the full CP-conserving 2HDM model, one can do so with the certainty that the λ_6 and λ_7 parameters will not spoil the restrictive bounds obtained in simpler models.

Finally, the bounds we obtained here considered that the stability and triviality conditions held at all scales between M_Z and M_U . More to the point, this procedure has the underlying assumption that the two-Higgs doublet model constitutes the whole of physics up to the gauge unification scale. That philosophy, however, can be readily inverted. A possible scenario is that within the next few years several scalar particles are discovered at the LHC, and that their properties conform to the 2HDM. However, suppose their masses completely violate all bounds presented here. This would, of course, suggest the existence of more new physics beyond the 2HDM but below the gauge unification scale, to justify the breaking of the stability and triviality bounds. The simplest example of such new physics would be the existence of a heavy fourth family of fermions, the presence of which would significantly change the form of the β -functions of the model, by virtue of the (necessarily large) associated Yukawa couplings.

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A The one loop β -functions

In this appendix we describe the calculation of the one loop β -functions for the general two Higgs scalar model defined by the potential given in Eq. (1). The calculation is straightforward by normal diagrammatic methods, and has already been presented in Ref. [3]. Here we describe an alternative algebraic calculation based on the renormalisation group equation satisfied by the effective potential, which we hope may be of some pedagogic interest, as well as providing a check on the previous calculation; in which we indeed thereby identify one fairly obvious typo. This procedure uses the RG invariance of the one loop effective potential, where all the β -functions may be computed from $S\text{Tr}M^4$, where M^2 is the mass matrix of the fields, including arbitrary vevs for the scalars. The basic procedure is explained in section 6 of reference [24].

Up to one loop, the effective potential $V_{\text{eff}}(\phi)$ for any theory is given by:

$$V_{\text{eff}}(\phi) = V(\phi) + V_1(\phi) + \dots \quad (25)$$

where $V(\phi)$ is the tree potential given in our case by Eq. (1), and

$$V_1(\phi) = \frac{\kappa}{4} \text{STr} M^4 \ln \frac{M^2}{\mu^2} \quad , \quad (26)$$

where the mass matrix M^2 includes contributions to all scalar, fermion and vector boson mass matrices with arbitrary background values of all scalar fields ϕ , $\kappa = (16\pi^2)^{-1}$ and STr is the usual spin-weighted trace.

In the Landau gauge, V_{eff} obeys the following RG equation:

$$\left[\mu \frac{\partial}{\partial \mu} + \sum_i \beta_i \frac{\partial}{\partial \lambda_i} - (\phi \gamma \frac{\partial}{\partial \phi} + \text{c.c.}) \right] V_{\text{eff}} = 0 \quad (27)$$

where the λ_i include all mass parameters and coupling constants, and γ is the matrix of anomalous dimensions of the scalar fields.

It follows that

$$\mathcal{D}^{(1)} V = -\mu \frac{\partial}{\partial \mu} V_1 = \frac{\kappa}{2} \text{STr} M^4 \quad , \quad (28)$$

where

$$\mathcal{D}^{(n)} = \sum_i \beta_i^{(n)} \frac{\partial}{\partial \lambda_i} - \left(\phi \gamma^{(n)} \frac{\partial}{\partial \phi} + \text{c.c.} \right). \quad (29)$$

By comparing coefficients of the various ϕ^4 terms on the two sides of Eq. (28) we can, if we know γ , determine all the one-loop β functions. From now on we write $\beta_i^{(1)} = \kappa \beta_i$ and $\gamma^{(1)} = \kappa \gamma$ to avoid writing factors of κ .

Let us first consider the simplified case when we set gauge and Yukawa couplings to zero. Then we can write M^2 as follows:

$$M^2 = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \quad (30)$$

where

$$A = \begin{pmatrix} \frac{\partial^2 V}{\partial \phi_i \partial \phi^{\dagger j}} & \frac{\partial^2 V}{\partial \phi_i \partial \xi^{\dagger j}} \\ \frac{\partial^2 V}{\partial \xi_i \partial \phi^{\dagger j}} & \frac{\partial^2 V}{\partial \xi_i \partial \xi^{\dagger j}} \end{pmatrix} \quad (31)$$

$$B = \begin{pmatrix} \frac{\partial^2 V}{\partial \phi_i \partial \phi_j} & \frac{\partial^2 V}{\partial \phi_i \partial \xi_j} \\ \frac{\partial^2 V}{\partial \xi_i \partial \phi_j} & \frac{\partial^2 V}{\partial \xi_i \partial \xi_j} \end{pmatrix} \quad (32)$$

$$C = \begin{pmatrix} \frac{\partial^2 V}{\partial \phi^{\dagger i} \partial \phi^{\dagger j}} & \frac{\partial^2 V}{\partial \phi^{\dagger i} \partial \xi^{\dagger j}} \\ \frac{\partial^2 V}{\partial \xi^{\dagger i} \partial \phi^{\dagger j}} & \frac{\partial^2 V}{\partial \xi^{\dagger i} \partial \xi^{\dagger j}} \end{pmatrix} \quad (33)$$

$$D = \begin{pmatrix} \frac{\partial^2 V}{\partial \phi^{\dagger i} \partial \phi_j} & \frac{\partial^2 V}{\partial \phi^{\dagger i} \partial \xi_j} \\ \frac{\partial^2 V}{\partial \xi^{\dagger i} \partial \phi_j} & \frac{\partial^2 V}{\partial \xi^{\dagger i} \partial \xi_j} \end{pmatrix} \quad (34)$$

and $i, j = 1, 2$ are $SU(2)$ indices, and to control the profusion of indices we have put $\Phi_1 \equiv \phi$ and $\Phi_2 \equiv \xi$.

It is then straightforward to write down the matrices A, B, C . Thus, for example,

$$\begin{aligned} (A_{11})^i_j &= \delta^i_j \left(\lambda_1 \phi^\dagger \phi + \lambda_3 \xi^\dagger \xi + \lambda_6 (\phi^\dagger \xi + \xi^\dagger \phi) \right) \\ &+ \lambda_1 \phi^{\dagger i} \phi_j + \lambda_4 \xi^{\dagger i} \xi_j + \lambda_6 (\phi^{\dagger i} \xi_j + \xi^{\dagger i} \phi_j) \end{aligned} \quad (35)$$

Since we are neglecting gauge and Yukawa terms we have no contribution to γ , we have from Eq. (28) that

$$\begin{aligned} \frac{1}{2} \text{STr} M^4 &= \frac{1}{2} (\text{Tr} A^2 + 2 \text{Tr} BC + \text{Tr} D^2) \\ &= \frac{\beta_{\lambda_1}}{2} (\phi^\dagger \phi)^2 + \frac{\beta_{\lambda_2}}{2} (\xi^\dagger \xi)^2 + \beta_{\lambda_3} (\phi^\dagger \phi) (\xi^\dagger \xi) + \beta_{\lambda_4} (\phi^\dagger \xi) (\xi^\dagger \phi) \\ &+ \left\{ \frac{\beta_{\lambda_5}}{2} (\phi^\dagger \xi)^2 + [\beta_{\lambda_6} (\phi^\dagger \phi) + \beta_{\lambda_7} (\xi^\dagger \xi)] (\phi^\dagger \xi + \text{h.c.}) \right\} . \end{aligned} \quad (36)$$

It is straightforward algebra to obtain from Eq. (36) that

$$\begin{aligned} \beta_{\lambda_1} &= 12\lambda_1^2 + 4\lambda_3^2 + 4\lambda_3\lambda_4 + 2\lambda_4^2 + 2\lambda_5^2 + 24\lambda_6^2 \\ \beta_{\lambda_2} &= 12\lambda_2^2 + 4\lambda_3^2 + 4\lambda_3\lambda_4 + 2\lambda_4^2 + 2\lambda_5^2 + 24\lambda_7^2 \\ \beta_{\lambda_3} &= (\lambda_1 + \lambda_2)(6\lambda_3 + 2\lambda_4) + 4\lambda_3^2 + 2\lambda_4^2 + 2\lambda_5^2 + 4\lambda_6^2 + 16\lambda_6\lambda_7 + 4\lambda_7^2 \\ \beta_{\lambda_4} &= 2(\lambda_1 + \lambda_2)\lambda_4 + 8\lambda_3\lambda_4 + 4\lambda_4^2 + 8\lambda_5^2 + 10\lambda_6^2 + 4\lambda_6\lambda_7 + 10\lambda_7^2 \\ \beta_{\lambda_5} &= 2(\lambda_1 + \lambda_2)\lambda_5 + 8\lambda_3\lambda_5 + 12\lambda_4\lambda_5 + 10\lambda_6^2 + 4\lambda_6\lambda_7 + 10\lambda_7^2 \\ \beta_{\lambda_6} &= 12\lambda_1\lambda_6 + 6\lambda_3(\lambda_6 + \lambda_7) + 8\lambda_4\lambda_6 + 4\lambda_4\lambda_7 + 10\lambda_5\lambda_6 + 2\lambda_5\lambda_7 \\ \beta_{\lambda_7} &= 12\lambda_2\lambda_7 + 6\lambda_3(\lambda_6 + \lambda_7) + 4\lambda_4\lambda_6 + 8\lambda_4\lambda_7 + 2\lambda_5\lambda_6 + 10\lambda_5\lambda_7. \end{aligned} \quad (37)$$

We have verified the above results by a standard Feynman diagram calculation. Moreover they are in full agreement with the results presented in Appendix A of Ref. [3], except for a typo in the result for β_{λ_2} there; the contribution $12\lambda_6^2$ there should read $12\lambda_7^2$. (Note that there is an overall difference of a factor of 2 between the definitions of all the β -functions).

The contributions to $\text{STr} M^4$ of $O(h^4)$ and $O(g^4, g^2 g'^2, g'^4)$ are easily calculated. The $O(g^4, g^2 g'^2, g'^4)$ terms come from the gauge boson mass matrix:

$$M_V^2 = \begin{pmatrix} \frac{1}{4} g^2 \phi^\dagger \{ \tau^a, \tau^b \} \phi + (\phi \rightarrow \xi) & \frac{1}{2} g g' \phi^\dagger \tau^a \phi + (\phi \rightarrow \xi) \\ \frac{1}{2} g g' \phi^\dagger \tau^a \phi + (\phi \rightarrow \xi) & \frac{1}{2} g'^2 \phi^\dagger \phi + (\phi \rightarrow \xi) \end{pmatrix} . \quad (38)$$

Then using the identities

$$\begin{aligned} \{ \tau^a, \tau^b \} &= 2\delta^{ab} \\ (\tau^a)_j^i (\tau^b)_l^k &= 2\delta_l^i \delta_j^k - \delta_j^i \delta_l^k \end{aligned} \quad (39)$$

one easily shows that

$$\begin{aligned} \text{STr} M_V^4 &= 3 \left(\frac{3}{4} g^4 + \frac{1}{4} g'^4 \right) \left((\phi^\dagger \phi)^2 + (\xi^\dagger \xi)^2 + 2\phi^\dagger \phi \xi^\dagger \xi \right) \\ &+ \frac{3}{2} g^2 g'^2 \left((\phi^\dagger \phi)^2 + (\xi^\dagger \xi)^2 + 4\phi^\dagger \xi \xi^\dagger \phi - 2\phi^\dagger \phi \xi^\dagger \xi \right) , \end{aligned} \quad (40)$$

where the STr has contributed a spin factor of 3.

The $O(h_t^4)$ contributions come from the top mass matrix:

$$M_t^2 = h_t^2 \phi^\dagger \phi \quad (41)$$

so that

$$\text{STr}M_t^4 = -12h_t^4(\phi^\dagger\phi)^2 \quad (42)$$

where the -12 consists of a colour factor of 3 and a spin factor of -4 .

The remaining contributions of $O(\lambda_i h_t^2, \lambda_i g^2, \lambda_i g'^2)$ come from the anomalous dimension term in Eq. (28), the anomalous dimensions in the Landau gauge being

$$\begin{aligned} \gamma_\phi &= 3h_t^2 - \frac{9}{4}g^2 - \frac{3}{4}g'^2 \\ \gamma_\xi &= -\frac{9}{4}g^2 - \frac{3}{4}g'^2 \end{aligned} \quad (43)$$

Armed with these results one easily shows from Eqs. (40),(42),(43) that Eq. (37) receives the following additional contributions:

$$\begin{aligned} \beta_{\lambda_1} &\rightarrow \beta_{\lambda_1} + \frac{3}{4}(3g^4 + g'^4 + 2g^2g'^2) - 3\lambda_1(3g^2 + g'^2 - 4h_t^2) - 12h_t^4 \\ \beta_{\lambda_2} &\rightarrow \beta_{\lambda_2} + \frac{3}{4}(3g^4 + g'^4 + 2g^2g'^2) - 3\lambda_2(3g^2 + g'^2) \\ \beta_{\lambda_3} &\rightarrow \beta_{\lambda_3} + \frac{3}{4}(3g^4 + g'^4 - 2g^2g'^2) - 3\lambda_3(3g^2 + g'^2 - 2h_t^2) \\ \beta_{\lambda_4} &\rightarrow \beta_{\lambda_4} + 3g^2g'^2 - 3\lambda_4(3g^2 + g'^2 - 2h_t^2) \\ \beta_{\lambda_5} &\rightarrow \beta_{\lambda_5} - 3\lambda_5(3g^2 + g'^2 - 2h_t^2) \\ \beta_{\lambda_6} &\rightarrow \beta_{\lambda_6} - 3\lambda_6(3g^2 + g'^2 - 3h_t^2) \\ \beta_{\lambda_7} &\rightarrow \beta_{\lambda_7} - 3\lambda_7(3g^2 + g'^2 - h_t^2). \end{aligned} \quad (44)$$

We also require the one-loop β -functions for g, g', g_3 and h_t , which are given by

$$\begin{aligned} \beta_{g'} &= 7g'^3 \\ \beta_g &= -3g^3 \\ \beta_{g_3} &= -7g_3^3 \\ \beta_{h_t} &= h_t \left[\frac{9}{2}h_t^2 - \frac{17}{12}g'^2 - \frac{9}{4}g^2 - 8g_3^2 \right]. \end{aligned} \quad (45)$$

B Renormalization group invariance and the exceptional region of parameter space

As mentioned in section 2, one can impose various symmetries on the 2HDM in order to obtain interesting physical consequences. It was recently proved [28] that the 2HDM potential can in fact only possess six distinct symmetries (including both discrete and continuous symmetries). This statement, however, hides a problem: since physical predictions cannot depend on the basis chosen for the Higgs doublets, the form of the potential - its specific combination of parameters and respective values - is not uniquely determined. For instance, we discussed the 2HDM potential with a Z_2 symmetry; nevertheless, a potential with a permutation symmetry between Φ_1 and Φ_2 - that is, a 2HDM potential invariant under the transformation $\Phi_1 \leftrightarrow \Phi_2$ - has exactly the same physical predictions. The reason is that both potentials, and both symmetries, are related by a basis change on the scalar doublets (see, for instance, [29]).

The question then arises, how does one know whether a potential has a given symmetry, since that symmetry can appear in an endless number of ways in different bases? The answer is, one builds basis-invariant quantities, the values of which reveal which, if any, symmetries the

potential has. There has been considerable attention to developing techniques to build basis invariants [30], first as a means of detecting CP violation and more recently to detect other types of symmetries [31, 27], such as $U(1)$ or Z_2 . In fact, the authors of Ref. [31] built a set of basis-invariant quantities which can distinguish between these two symmetries. However, that method failed to identify the presence of a continuous symmetry for a given combination of parameters, the so-called exceptional region of parameter space (ERPS): namely,

$$\begin{aligned} m_{22}^2 &= m_{11}^2, & m_{12}^2 &= 0, \\ \lambda_2 &= \lambda_1, & \lambda_7 &= -\lambda_6 \quad . \end{aligned} \quad (46)$$

Of particular relevance is the fact that the ERPS is not a zero-measure set of parameters, but is itself instead attained through the imposition, on the potential, of a given symmetry [31]. In fact, several possible symmetries lead into the ERPS: either combinations of discrete symmetries or generalised CP symmetries (generalised in the sense that they do not satisfy $\text{CP}^2 = 1$). In fact, in [27] two generalised CP symmetries were identified, which lead into the ERPS: one was a discrete CP symmetry (dubbed CP2 in that reference), the other a continuous one (dubbed CP3). The problem of identifying the presence of a continuous symmetry was solved in Ref. [27], with a new basis invariant quantity D , which is written, in the ERPS, in a basis where all potential parameters are real, as

$$D = -\frac{1}{27} [\lambda_5(\lambda_1 - \lambda_3 - \lambda_4 + \lambda_5) - 2\lambda_6^2]^2 [(\lambda_1 - \lambda_3 - \lambda_4 - \lambda_5)^2 + 16\lambda_6^2] . \quad (47)$$

As shown in [27], once in the ERPS the basis-invariant condition $D = 0$ indicates the presence of a continuous $U(1)$ symmetry. If $\lambda_6 = 0$, then $D = 0$ gives the following conditions:

$$\lambda_5 = 0, \quad \lambda_5 = \pm(\lambda_1 - \lambda_3 - \lambda_4). \quad (48)$$

If $\lambda_6 \neq 0$, then $D = 0$ corresponds to

$$2\lambda_6^2 = \lambda_5(\lambda_1 - \lambda_3 - \lambda_4 + \lambda_5). \quad (49)$$

In Ref. [27] the RG invariance of the $D = 0$ condition itself was demonstrated, through a calculation which managed to avoid using the explicit form of the λ_i β -functions. Using the explicit form of the β -functions of Appendix A, it is simple to demonstrate that:

- The conditions on the quartic couplings that define the ERPS are RG invariant. Explicitly, we see that:
 - If $\lambda_1 = \lambda_2$ and $\lambda_6 = -\lambda_7$, then we will have $\beta_{\lambda_1} = \beta_{\lambda_2}$ and $\beta_{\lambda_6} = -\beta_{\lambda_7}$.
- Each of the conditions in Eqs. (48) and (49) are RG invariant, if we are in the ERPS (that is, with $\lambda_2 = \lambda_1$ and $\lambda_7 = -\lambda_6$). Namely,
 - If $\lambda_5 = \lambda_6 = 0$, then $\beta_{\lambda_5} = \beta_{\lambda_6} = 0$;
 - If $\lambda_6 = 0$ and $\lambda_5 = \pm(\lambda_1 - \lambda_3 - \lambda_4)$, then we have $\beta_{\lambda_5} = \pm(\beta_{\lambda_1} - \beta_{\lambda_3} - \beta_{\lambda_4})$ and $\beta_{\lambda_6} = 0$;
 - Finally, and much in the same manner, if $\xi = 2\lambda_6^2 - \lambda_5(\lambda_1 - \lambda_3 - \lambda_4 + \lambda_5) = 0$ then we also have $\beta_\xi = 0$.

However, there is a detail which must be mentioned: the above is true if one sets the Yukawa coupling h_t equal to zero - that is, if the theory does not couple to fermions (the *gauge coupling* contributions from Eq. (44) may be included, however). In fact, it is extremely difficult, if not impossible, to couple the two doublets to fermions in a phenomenological acceptable manner, if the symmetries which lead to the ERPS are in place. In Ref. [32], for instance, the authors managed to couple the fermion sector and the scalar one in the presence of a CP2 symmetry, but those couplings implied masslessness for the two first generations.

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